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Code Number 041/1/3



**INDIAN SCHOOL MUSCAT
THIRD PRELIMINARY EXAMINATION
MATHEMATICS**

CLASS: XII

Sub. Code: 041

Time Allotted: 3 Hrs

01.02.2018

SET-3

Max. Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION – A (4 x 1 = 4)
Questions 1 to 4 carry 1 mark each

1. Write the adjoint of $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$ 1
2. Find the points where the function $f(x) = [x]$, $1/2 \leq x < 3$ is not differentiable. 1
3. Find $\frac{dy}{dx}$, if $y = \tan^{-1}(\cot x)$ 1
4. Find the Cartesian equation of the line that passes through the points $(3,-2,-5)$, $(3,-2,6)$ 1

SECTION – B (8 x 2 = 16)
Questions 5 to 12 carry 2 marks each

5. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, find $\frac{dy}{dx}$. 2
6. A balloon which is always remains spherical, has a variable radius. Find the rate at which its volume is increasing w.r.t its radius when the radius is 7cm. 2
7. Evaluate: $\int (\log x) dx$ 2
8. Find the points on the curve $y = x^3 - 11x + 5$ at which the tangent has the equation $y = x - 11$ 2

9. Find a unit vector perpendicular to each of the vector 2
 $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.
10. Ten eggs are drawn successively with replacement from a lot containing 5% defective eggs. Find the probability that there is exactly one defective egg. 2
11. There are two types of fertilizers F_1 and F_2 . F_1 consists of 10% of nitrogen and 6% phosphoric acid and F_2 consists of 5% of nitrogen and 10% phosphoric acid. After testing the soil conditions a farmer finds that she needs at least 14 kg of nitrogen and 14kg of phosphoric acid for her crops, If F_1 costs Rs6/kg and F_2 costs Rs5/Kg. Formulate the problem so that nutrient requirements are met at a minimum cost. 2

12. Find the angle between the pair of the lines given by 2
 $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

SECTION – C (11 x 4 = 44)
 Questions 13 to 23 carry 4 marks each

13. If $y = \sqrt{\frac{(x-3)(x^2+3)}{3x^2+4x+5}}$, find $\frac{dy}{dx}$. 4

OR

Find the value of k, if $f(x) = \begin{cases} \frac{1-\cos 2x}{3x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ at $x = 0$.

14. Solve The Following: $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ 4
15. Integrate: $\int \frac{3x-2}{(x+1)^2(x+3)} dx$. 4
16. Evaluate the integral: $\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$. 4
 (OR)

Evaluate the integral: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$.

17. If with reference to the right handed system of mutually perpendicular unit vectors \hat{i}, \hat{j} and \hat{k} , $\vec{a} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to \vec{a} and $\vec{\beta}_2$ is perpendicular to \vec{a} . 4
18. Solve :- $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ given $y = 0$ when $x = 1$ 4

19. Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$ 4

OR

Prove that $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

20. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} & $\vec{c} \cdot \vec{d} = 15$ 4

21. In an examination, an examinee either guesses or copies or knows the answer of multiple choice question with 3 choice. The probability that he makes a guess is $1/3$ and the probability he copies is $1/6$. Find the probability that his answer is correct, given that he copied it is $1/8$. Find the probability that he knew the answer to the question, given that he correctly answered it. 4

22. Two Cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of kings. Also find the mean. 4

23. Maximise $Z = 3x + 5y$, subject to the constraints: 4
 $x + 2y \leq 20$, $x + y \leq 15$, $y \leq 5$, $x, y \geq 0$.

SECTION – D (6 x 6 = 36)
 Questions 24 to 29 carry 6 marks each

24. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. 6

OR

The length of the sides of an isosceles triangle are $9+x^2$, $9+x^2$ and $18-2x^2$ units. Calculate the value of x which makes the area maximum. Also find the maximum area of the triangle.

25. The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12. Using matrices, find numbers. 6

26. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$ 6
 OR
 Using integration, find the area of the following region: $\{(x,y): |x - 1| \leq y \leq \sqrt{5 - x^2}\}$

27. Solve:- $\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$ 6

28. Find the equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. 6

OR

Find the length and the foot of the perpendicular from the point $(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$, also find image point.

29. Let $f: \mathbf{R}_+ \rightarrow [-5, \infty]$ be a function defined as $f(x) = 9x^2 + 6x - 5$. show that 6

f is invertible. Find the inverse of f .

OR

Let X be a non empty set. $P(X)$ be its power set. Let $*$ be an operation defined on elements of $P(X)$, by: $A * B = A \cap B$ for all $A, B \in P(X)$. Then

- a) Prove that $*$ is a binary operation on $P(X)$.
- b) Is $*$ commutative?
- c) Is $*$ associative?
- d) Find the identity element in $P(X)$ w.r.t. $*$.
- e) Find all invertible element of $P(X)$.
- f) If \blacksquare is another binary operation defined on $P(X)$ as $A \blacksquare B = A \cup B$, then verify that \blacksquare distributes itself over $*$.

End of the Question Paper